Fórmulas de Mecánica Clásica

1. Mecánica Newtoniana

$\boldsymbol{p} = m\dot{\boldsymbol{x}}$ *Momento lineal*

$\sum_i \boldsymbol{F}_i = \dot{\boldsymbol{p}}$ *Ley de Newton*

$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ *Momento angular*

$\sum_i \boldsymbol{\tau}_i = \dot{\boldsymbol{L}}$ *Torque*

$T = \frac{1}{2}m\|\boldsymbol{v}\|^2$ *Energía cinética*

$\boldsymbol{F} = -\boldsymbol{\nabla}V$ *Energía potencial*

$W = \int \boldsymbol{F}d\boldsymbol{x}$ *Trabajo*

$E = T + V$ *Energía*

2. Mecánica Langrangiana

$\sum_i (\boldsymbol{F}_i^a - \dot{\boldsymbol{p}}_{\boldsymbol{i}}) \cdot \delta\boldsymbol{r}_i = 0$ *Principio de D'Alembert*

$Q = \sum_i \boldsymbol{F}_{\boldsymbol{i}} \cdot \frac{\partial \boldsymbol{r}_{\boldsymbol{i}}}{\partial q}$ *Fuerza generalizada*

$\mathcal{L} = T - V$ *Lagrangiano*

$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = 0$ *Ecuación de Euler − Lagrange*

$V_{EM} = q(V - \frac{1}{c}\boldsymbol{v} \cdot \boldsymbol{A})$ *Potencial electromagnético*

3. Principios Variacionales y Simetrías

$S = \int \mathcal{L}dt$ *Acción*

$p = \frac{\partial \mathcal{L}}{\partial \dot{q}}$ *Momento generalizado*

$\sum_i (k_i p_i) - \tau t = cte.$ *Teorema de Noether*

$H = \sum_i (\dot{q}_i p_i) - \mathcal{L}$ *Función de Hamilton*

4. Fuerzas Centrales

$$\boldsymbol{R} = \frac{\sum_i m_i \boldsymbol{r}_i}{\sum_i m_i} \qquad \textit{Centro de masa}$$

$$M = \sum_i m_i \qquad \textit{Masa total}$$

$$\boldsymbol{r} = \boldsymbol{r}_i - \boldsymbol{r}_j \qquad \textit{Posición relativa}$$

$$\mu = \left(\sum_i \frac{1}{m_i}\right)^{-1} \qquad \textit{Masa reducida}$$

$$\mathcal{L} = \frac{1}{2}M\left\|\dot{\boldsymbol{R}}\right\|^2 + \frac{1}{2}\mu\|\dot{\boldsymbol{r}}\|^2 - V$$

$$A = \frac{1}{2}\dot{\theta}r^2 \qquad \textit{Velocidad aerolar}$$

$$\dot{A} = 0 \qquad \textit{Segunda Ley de Kepler}$$

$$V_{eff} = \frac{{L_z}^2}{2\mu r^2} + V \qquad \textit{Potencial efectivo}$$

$$\theta = \int_{r_0}^{r} \frac{L_z}{\mu r^2} \frac{dr}{\sqrt{\frac{2}{\mu}(E - V_{eff})}} + \theta_0 \qquad \textit{Ecuación de órbita}$$

$$\frac{d^2(\frac{1}{r})}{d\theta^2} + \frac{1}{r} = \frac{\mu}{{L_z}^2\left(\frac{1}{r}\right)^2} V_{(\frac{1}{r})} \qquad \textit{Ecuación diferencial de órbita}$$

$$\epsilon = \sqrt{1 - \frac{E}{E_c}} \qquad \textit{Excentricidad}$$

$$\epsilon_e = \sqrt{1 - \frac{b^2}{a^2}} \qquad \textit{Excentricidad de la elipse}$$

$$r = \frac{r_c}{1 + \epsilon\cos(\theta)} \qquad \textit{Ecuación de las cónicas}$$

$$T^2 = \frac{4\pi^2}{GM}a^3 \qquad \textit{Tercera Ley de Kepler}$$

5. Pequeñas Oscilaciones

$$\epsilon_j = q_j - q_j^{eq}$$

$$V_{jk} = \frac{\partial^2 V}{\partial q_j \partial q_k}$$

$$M_{jk} = \sum_i m_i \frac{\partial \boldsymbol{r}_i}{\partial q_j} \cdot \frac{\partial \boldsymbol{r}_i}{\partial q_k}$$

$$V = \sum_{j,k} \frac{1}{2} V_{jk}\epsilon_j\epsilon_k \qquad \textit{Potencial Elástico}$$

$$\mathcal{L} = \sum_{j,k}\left(\frac{1}{2}M_{jk}\dot{\epsilon}_j\dot{\epsilon}_k - \frac{1}{2}V_{jk}\epsilon_j\epsilon_k\right) \equiv \frac{1}{2}\dot{\boldsymbol{\epsilon}}^\dagger \cdot \mathbb{M} \cdot \dot{\boldsymbol{\epsilon}} - \frac{1}{2}\boldsymbol{\epsilon}^\dagger \cdot \mathbb{V} \cdot \boldsymbol{\epsilon}$$

$$\sum_k (M_{jk}\ddot{\epsilon}_j + V_{jk}\epsilon_j) = 0 \equiv \mathbb{M} \cdot \ddot{\boldsymbol{\epsilon}} + \mathbb{V} \cdot \boldsymbol{\epsilon} = 0$$

$$\mathbb{A}^{-1} = \mathbb{A}^{\dagger}$$

$$\mathbb{\Omega} = \mathbb{A}^{\dagger} \cdot \mathbb{V} \cdot \mathbb{A}$$

$$\boldsymbol{\zeta} = \mathbb{A}^{\dagger} \cdot \boldsymbol{\epsilon}$$

$$\mathbb{A}^{\dagger} \cdot \mathbb{M} \cdot \mathbb{A} = \mathbb{1}$$

$$\mathcal{L} = \frac{1}{2}\dot{\boldsymbol{\zeta}}^{\dagger} \cdot \dot{\boldsymbol{\zeta}} - \frac{1}{2}\dot{\boldsymbol{\zeta}}^{\dagger} \cdot \mathbb{V} \cdot \dot{\boldsymbol{\zeta}} \qquad \textit{Lagrangiano desacoplado}$$

6. Cuerpo Rígido

$$\mathbb{A} = \begin{pmatrix} \cos(\phi)\cos(\psi) - \sin(\phi)\cos(\theta)\sin(\psi) & \sin(\phi)\cos(\psi) + \cos(\phi)\cos(\theta)\cos(\psi) & \sin(\theta)\sin(\psi) \\ -\cos(\phi)\sin(\psi) - \sin(\phi)\cos(\theta)\cos(\psi) & -\sin(\phi)\sin(\psi) + \cos(\phi)\cos(\theta)\cos(\psi) & \sin(\theta)\cos(\psi) \\ \sin(\phi)\sin(\theta) & -\cos(\phi)\sin(\theta) & \cos(\theta) \end{pmatrix}$$

Matriz cambio de base

$$\boldsymbol{v}_p = \boldsymbol{v}_o + \boldsymbol{\Omega} \times \boldsymbol{r}_{op} \qquad \textit{Ecuación cinemática del cuerpo rígido}$$

$$I_{jk} = \iiint_{\Xi} \rho_{(\boldsymbol{r})} \left(\|\boldsymbol{r}\|^2 \delta_{jk} - r_j r_k \right) d\Xi \qquad \textit{Tensor de Inercia}$$

$$T = \frac{1}{2} M \left\| \dot{\boldsymbol{R}} \right\|^2 + \frac{1}{2} \boldsymbol{\Omega}^{\dagger} \cdot \mathbb{I} \cdot \boldsymbol{\Omega} \qquad \textit{Energía cinética del cuerpo rígido}$$

$$I^p_{jk} = MR^2 \delta_{jk} - MR_j R_k + I^{CM}_{jk} \qquad \textit{Teorema de Steiner}$$

$$\boldsymbol{L}^{CM} = \mathbb{I} \cdot \boldsymbol{\Omega} \qquad \textit{Spin}$$

$$\boldsymbol{L}^o = \boldsymbol{R} \times \boldsymbol{p} + \boldsymbol{L}^{CM} \qquad \textit{Momento angular del cuerpo rígido}$$

$$I_1 \dot{\Omega}_1 + (I_3 - I_2)\Omega_2 \Omega_3 = \tau_1$$

$$I_2 \dot{\Omega}_2 + (I_1 - I_3)\Omega_1 \Omega_3 = \tau_2 \qquad \textit{Ecuaciones de Euler}$$

$$I_3 \dot{\Omega}_3 + (I_2 - I_1)\Omega_1 \Omega_2 = \tau_3$$

7. Formulación Hamiltoniana

$$\mathcal{H} = \sum_j \dot{q}_j p_j - \mathcal{L} \qquad \textit{Hamiltoniano}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}} = p \ , \ \frac{\partial \mathcal{L}}{\partial \dot{p}} = 0 \ , \ \frac{\partial \mathcal{L}}{\partial q} = -\frac{\partial \mathcal{H}}{\partial q} = \dot{p} \ , \ \frac{\partial \mathcal{L}}{\partial p} = \dot{q} - \frac{\partial \mathcal{H}}{\partial p} = 0 \ , \ \frac{\partial \mathcal{L}}{\partial t} = -\frac{\partial \mathcal{H}}{\partial t}$$

Ecuaciones canónicas de Hamilton

$$\frac{\partial \mathcal{H}}{\partial t} = \frac{d\mathcal{H}}{dt}$$

$$\dot{Q} = \frac{\partial \mathcal{K}}{\partial P}, \ \dot{P} = -\frac{\partial \mathcal{K}}{\partial Q} \qquad \textit{Transformaciones canónicas}$$

$$F = F_{1(q,Q,t)} \implies p = \frac{\partial F}{\partial q}, \ P = -\frac{\partial F}{\partial Q}, \ \mathcal{K} = \mathcal{H} + \frac{\partial F}{\partial t}$$

$$F = F_{2(q,P,t)} - \sum_i Q_i P_i \implies p = \frac{\partial F}{\partial q}, \ Q = \frac{\partial F}{\partial P}, \ \mathcal{K} = \mathcal{H} + \frac{\partial F}{\partial t}$$

$$F = F_{3(Q,P,t)} + \sum_i q_i p_i \implies q = -\frac{\partial F}{\partial p}, \ p = \frac{\partial F}{\partial Q}, \ \mathcal{K} = \mathcal{H} + \frac{\partial F}{\partial t}$$

$$F = F_{4(p,P,t)} + \sum_i (q_i p_i - Q_i P_i) \implies q = -\frac{\partial F}{\partial p},\ Q = \frac{\partial F}{\partial P},\ \mathcal{K} = \mathcal{H} + \frac{\partial F}{\partial t}$$

$$\mathbb{J} = \begin{pmatrix} \mathbb{0} & \mathbb{1} \\ -\mathbb{1} & \mathbb{0} \end{pmatrix}$$

$$\mathbb{M} \cdot \mathbb{J} \cdot \mathbb{M}^\dagger = \mathbb{J} = \mathbb{M}^\dagger \cdot \mathbb{J} \cdot \mathbb{M}$$

$$\det(\mathbb{M})^2 = 1$$

$$[U, V] = \sum_i \left(\frac{\partial U}{\partial q_i} \frac{\partial V}{\partial p_i} - \frac{\partial U}{\partial p_i} \frac{\partial V}{\partial q_i} \right)$$

$$[U, [V, W]] + [V, [W, U]] + [W, [U, V]] = 0$$

$$\dot{q} = [q, \mathcal{H}],\ \dot{p} = [p, \mathcal{H}]$$

$$f_{(t)} = f e^{\hat{G} t} = f_{(0)} + [f, G]|_{t=0}\ t + \frac{1}{2}[[f, G], G]|_{t=0}\ t^2 + ...$$

$$\mathcal{H}_{(q, \frac{\partial S}{\partial q}, t)} + \frac{\partial S}{\partial t} = 0 \qquad Ecuación\ de\ Hamilton - Jacobi$$

$$W_{(q,P)} = S_{(q,P,t)} - P_1 t = \sum_i W_{(q_i,P)} \qquad Función\ característica\ de\ Hamilton$$

$$J = \frac{1}{2\pi} \oint pdq \qquad Variable\ de\ acción$$

$$\theta = \frac{\partial W}{\partial J} \qquad Variable\ de\ ángulo$$

$$\frac{\partial E}{\partial J} = \dot{\theta} = \omega,\ \dot{J} = 0,\ W = \int_{q_0}^{q} pdq$$

8. Relatividad Especial

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \|\boldsymbol{\beta}\|^2}} \qquad Factor\ de\ Lorentz$$

$$\boldsymbol{r'} = \boldsymbol{r} + (\gamma - 1) \frac{(\boldsymbol{\beta} \cdot \boldsymbol{r}) \cdot \boldsymbol{\beta}}{\|\boldsymbol{\beta}\|^2} - \gamma c t \boldsymbol{\beta}$$

$$t' = \gamma t - \frac{\gamma}{c} (\boldsymbol{\beta} \cdot \boldsymbol{r}) \qquad Transformación\ de\ Lorentz$$

$$x_\mu = (x, y, z, ict),\ x_\mu^2 = x^2 + y^2 + z^2 - c^2 t^2 \qquad Espacio - Tiempo$$

$$f_R = f_0 \gamma (1 - \beta) \qquad Efecto\ Doppler\ relativista$$

$$d\tau^2 = -\frac{dx_\mu^2}{c^2} = -\frac{dx'^{\,2}_\mu}{c^2} \qquad Tiempo\ propio$$

$$dt = \gamma d\tau \qquad Dilatación\ temporal$$

$$L = \frac{L_o}{\gamma} \qquad Contracción\ de\ longitud$$

$$U_\mu = \left(\frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}, ic\frac{dt}{d\tau} \right) \qquad Cuadrivector\ velocidad$$

$$J_\mu = (J_x, J_y, J_z, ic\rho) \qquad Cuadrivector\ densidad\ de\ corriente$$

$$\frac{dJ_\mu}{dx_\mu} = 0 \qquad Conservación\ de\ la\ carga$$

$A_\mu = \left(A_x, A_y, A_z, i\frac{V}{c} \right)$ *Cuadrivector potencial electromagnético*

$P_\mu = mU_\mu$ *Cuadrivector momento lineal*

$K_\mu = \frac{dP_\mu}{d\tau}$ *Fuerza de Minkowski*

$F_i = \frac{K_i}{\gamma}$

$E = mc^2$ *Energía en reposo*

$T = (\gamma - 1)mc^2 = \sqrt{m^2c^4 + P_i^2 c^2}$ *Energía cinética relativista*

$\mathcal{L} = -\frac{mc^2}{\gamma} - V$ *Lagrangiano relativista*

$S = -mc^2 \int d\tau$ *Acción relativista*